



Discounting in the presence of scarce ecosystem services[☆]

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ABSTRACT

The discount rate for cost-benefit analysis has to take account of future scarcity of ecosystem services in consumption and production. Previous literature focuses on the first aspect and shows the importance of the relative price effect, for given growth rates of consumption and ecosystem services. This paper focuses on intermediate ecosystem services in production and shows that for limited substitutability and a low growth rate of these ecosystem services, the growth rate of consumption, and thus the discount rate, declines towards a low value. Using a Ramsey growth model, the paper distinguishes three cases. If ecosystem services can be easily substituted, the discount rate converges to the usual value in the long term. Secondly, if ecosystem services can be easily substituted in production but not in consumption, the relative price effect is important. Finally, and most interestingly, if ecosystem services cannot be easily substituted in production, the discount rate declines towards a low value and the relative price effect is less important. Another part of the previous literature has shown that a declining discount rate is the result of introducing several forms of uncertainty, but this paper reaches that conclusion from an endogenous effect on the growth rate of the economy.

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1. Introduction

Discounting is the way in which we relate the future and the current costs and benefits in cost-benefit analyses for public policy and project evaluation. Since the pioneering work of Fisher and Krutilla (1975) and later Weitzman (1994), the literature has given a lot of attention recently to the role and the relative scarcity of the natural environment in the provision of ecosystem services (e.g., Guesnerie, 2004; Weikard and Zhu, 2005; Hoel and Sterner, 2007; Gollier, 2010; Traeger, 2011). An important argument is that the growth rates of the economy and the ecosystem services differ. This implies that the usual assumption of constant relative prices does not hold, since the valuation of environmental benefits relative to produced consumption goods changes over time. Cost-benefit analyses should either take the expected change in relative prices into account or use different discount rates for consumption and for ecosystem services. The elasticity of substitution plays, of

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course, an important role. This literature derives the discount rate from the Ramsey rule¹ with a reduced form for welfare depending on the flow of consumption and on the quality of the environment. An interesting result is that the change in relative prices, or the gap between the discount rates, is proportional to the difference between these growth rates, and inversely related to the elasticity of substitution. Empirical work (Baumgärtner et al., 2015; Koetse et al., 2018; Drupp, 2018) shows that the discount rate for ecosystem services is smaller than the discount rate for consumption, with a gap in the order of one percentage point and varying with the type and the role of the ecosystem service that is considered.

A reduced form for welfare with exogenous growth rates for consumption and for the quality of the environment does not take the different roles of ecosystem services into account. Ecosystem services can have a direct amenity value, but ecosystem services can also have an intermediate role as an input factor into production. It follows that the growth rate of consumption can depend on the availability of ecosystem services and on the possibilities of substitution. More specifically, technological progress may drive economic growth, but growth may be restricted if ecosystem services in production do not grow and cannot be easily substituted in production. This has an important effect on the discount rate. The literature on changing relative prices mostly assumes that the growth rates of consumption and ecosystem services are exogenous and given. Some of the papers consider a feedback effect of damage from emissions on the availability of ecosystem services (Sterner and Persson, 2008; Drupp and Hänsel, 2018), or consider joint stochastic dynamics of the growth rates of consumption and ecosystem services (Gollier, 2010), so that the growth rates are connected. This paper adds the endogenous effect of limited availability and substitutability of ecosystem services in production on the growth rate of the economy, and therefore on the discount rate. The literature on economic growth has shown that if the factors of production are complements, the long-run growth rate is limited by the growth rate of the scarcest factor (Acemoglu and Guerrieri, 2008; Quaas and Bröcker, 2016), but this has not been discussed in relation with the discount rate. Moreover, this paper shows that the effect on the discount rate of limited substitutability of ecosystem services in production is much stronger than the relative price effect that results from limited substitutability of ecosystem services directly in utility.

In order to analyse the roles of ecosystem services both in utility and in production, we use a Ramsey growth model. Intermediate ecosystem services are a factor of production and final ecosystem services are an argument in the welfare function. In order to allow for substitution, we use a constant elasticity of substitution (CES) utility function as well as a CES production function. We derive the steady-state conditions for balanced growth. If the elasticities of substitution are high, we get the standard growth rate and discount rate in the long run. However, if the elasticity of substitution in production is low, we get a low growth rate in the long run, with a low discount rate given by the Ramsey rule. If the elasticity of substitution is high in production but low in welfare, we get a high growth rate in the long run, but also the relative price effect that was considered in the extant literature. Moreover, the Ramsey model allows considering the paths of the growth rate and the discount rate towards the balanced-growth steady state. In this way, we can show the effects of the limited availability and substitutability of ecosystems services on the term structure of the discount rate.

An important conclusion is that when the restrictions of the natural environment start to kick in, the path of the discount rate is declining towards a very low level in the long run. The term structure of the discount rate will inherit the negative slope from the term structure of the growth rate (Gollier and Hammitt, 2014). A declining discount rate also results from various effects of uncertainty (Gollier, 2002, 2010, 2013; Newell and Pizer, 2003; Weitzman, 2007, 2010). Several countries have implemented this in their official policy (Groom and Hepburn, 2017). The main idea is that increasing uncertainty about the growth rate of the economy increases the effects of risk aversion and of prudence, due to concavity of utility and convexity of marginal utility. It is important to note that the literature on limited substitutability of ecosystem services in utility also yields non-constant discount rates (Traeger, 2011). More specifically, the discount rate is declining for values of the elasticity of substitution in utility in relation with the intertemporal elasticity of substitution. However, this paper shows, in an integrated framework with ecosystem services in both utility and production, that this effect is relatively small.

Our paper is complementary to studies that find a declining discount rate through other mechanisms. Dietz and Venmans (2019) find the result in the presence of habit formation and loss aversion. Brock and Xepapadeas (2018) show that greenhouse gas emissions and temperature change, yielding climate damage in the utility function, also yield a declining discount rate. We leave uncertainty and environmental damage out in this paper, but it is clear that all arguments point in the same direction: a lower discount rate in the long term. This is very important for cost-benefit analyses with a very long time horizon, such as the costs and benefits of climate change mitigation and adaptation. A constant discount rate effectively means that the benefits in the long run hardly count, but a declining discount rate changes the picture.

This paper provides a framework for analysing the effects on the discount rate that are caused by a possible low growth rate and limited substitutability of ecosystem services. It is, of course, an empirical question to determine the sectors for which this is relevant, and to quantify the effects by determining growth rates and elasticities of substitution.

Section 2 analyses the role of ecosystem services in production, using a Ramsey growth model. Section 3 adds the role of ecosystem services in utility, in order to get the full picture with the three different cases mentioned above, namely the cases with a high and low discount rate in the long run and the case with a high discount rate and the relative price effect. Section 4 provides numerical simulations to show the paths of the growth rate and the discount rate in the different cases. Section 5 concludes.

¹ The Ramsey rule yields the discount rate as the rate of pure time preference plus the product of the elasticity of marginal utility and the growth rate.

2. Ecosystem services in production

Ecosystems are natural capital. Services from ecosystems provide amenity value in the utility function, but they also provide essential inputs (e.g. pollination and water) into production processes and therefore also have production value. Ecosystems follow the dynamics of biophysical processes. Human activities influence the processes, but we abstract from this and take an exogenous growth rate g_E (positive or negative) for the ecosystems. Ecosystem services E are either a stock or a flow variable. For example, pollination that is important for agricultural production processes can be measured by the number of bees in some neighbouring natural area at some point in time, which is a stock variable. A change in land use changes the number of bees and thus the pollination capacity. However, extractions of water and other resources to be used in production are flow variables. We simply assume in this paper that the availability or the quality of E affects utility and production. We remain agnostic about which combination of externalities and (second-best) policies determine availability, and investigate how the constant rate g_E of growth (or de-growth) of E affects the growth rate of the economy.

We use a Ramsey growth model, with ecosystem services as an input in both the utility and production function. This allows us to consider the effect of intermediate ecosystem services on the growth rate of the economy and on the discount rate. The optimal allocation of investment and consumption is determined by maximizing the integral of discounted welfare U of consumption C and ecosystem services E over time, subject to the accumulation of capital K :

$$\begin{aligned} \max_{C(t)} \int_0^{\infty} e^{-\rho t} U(C(t), E(t)) dt, \\ \dot{K}(t) = F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \end{aligned} \quad (1)$$

where F denotes the (net) production function, E ecosystem services, and ρ the pure rate of time preference. Effective labour input H is labour input scaled by human capital and labour-augmenting technology. It grows at exogenous constant rate g_H . Ignoring population growth and human capital, we may simply refer to the growth rate g_H as the rate of technical change. In order to capture the increasing scarcity of the ecosystem services, we assume throughout the paper that $g_E < g_H$. Note that we do not assume a full-optimizing framework because the growth rates g_H and g_E are exogenously fixed; investment decisions are optimized conditional on a given, potentially sub-optimal, time path of ecosystem services.

The (consumption) discount factor D is the marginal contribution to welfare of future consumption relative to the marginal contribution to welfare of current consumption, or the marginal rate of substitution:

$$D(t) = \frac{e^{-\rho t} U_C(C(t), E(t))}{U_C(C(0), E(0))}. \quad (2)$$

The (consumption) discount rate r is therefore the rate at which the discount factor falls, $r(t) = -\dot{D}(t)/D(t)$, which implies:

$$r(t) = \rho - \frac{\dot{U}_C(C(t), E(t))}{U_C(C(t), E(t))}. \quad (3)$$

The discount rate reflects the additional minimum amount of consumption the society requires at time t , in exchange for giving up one unit of consumption at time $t - dt$ (with dt arbitrarily small) without suffering a decline in welfare.²

With the current - value Hamiltonian function $U(C, E) + \lambda(F(K, E, H) - C)$, optimal allocation in problem (1) requires

$$\begin{aligned} U_C(C, E) &= \lambda, \\ \dot{\lambda}(t) - \rho\lambda(t) &= -F_K(K(t), E(t), H(t))\lambda(t), \end{aligned} \quad (4)$$

where λ denotes the shadow value of capital and $F_K(K(t), E(t), H(t))$ the (net) marginal product of capital. Comparing (3) and (4), we see that optimality requires that the discount rate, r , equals the (net) marginal product of capital, F_K .

We want to allow for substitution between consumption and the amenity of ecosystem services in consumer utility, and therefore we follow [Hoel and Sterner \(2007\)](#) and use the CES intratemporal utility function nested into the constant relative risk aversion (CRRA) intertemporal utility:

$$U(C, E) = \frac{1}{1-\gamma} \left[(1-\pi)C^{\frac{\zeta-1}{\zeta}} + \pi E^{\frac{\zeta-1}{\zeta}} \right]^{\frac{(1-\gamma)\zeta}{\zeta-1}}, \quad (5)$$

² We can similarly define an environmental discount rate $asr^E(t) = \rho - \dot{U}_E(C(t), E(t))/U_E(C(t), E(t))$.

where γ ($\gamma \neq 1$) denotes the inverse of the elasticity of intertemporal substitution, and where π denotes the relative shares and ζ ($\zeta \neq 1$) the elasticity of substitution between consumption and ecosystem services in utility.

In this section, we take $\pi = 0$ and focus on the role of intermediate ecosystem services on the growth rate of the economy. The utility function reduces to $U(C) = C^{1-\gamma}/(1-\gamma)$ and substitution of the first part of (4) into the second part of (4) leads to the Keynes-Ramsey rule for the optimal consumption path

$$\dot{C}(t) = \gamma^{-1}(F_K(K(t), E(t), H(t)) - \rho)C(t), \quad (6)$$

where a transversality condition has to hold. In order to find the path of all the variables, we have to solve the system:

$$\begin{aligned} \dot{K}(t) &= F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \\ \dot{C}(t) &= \gamma^{-1}(F_K(K(t), E(t), H(t)) - \rho)C(t), \\ \dot{H}(t) &= g_H H(t), H(0) = H_0, \\ \dot{E}(t) &= g_E E(t), E(0) = E_0. \end{aligned} \quad (7)$$

The solution of (7) allows us to identify the time path of the discount rate $r = F_K$.

2.1. Cobb-Douglas production system with ecosystem services

For a Cobb-Douglas production function, it is easy to show that the economy converges to a steady state with balanced growth in which the growth rate g depends on the growth rates of technological change and ecosystem services. Suppose that the production function is given by

$$F^{CD}(K, E, H) = AK^\alpha E^\beta H^{1-\alpha-\beta}, \quad (8)$$

where A denotes total factor productivity and α , β and $1 - \alpha - \beta$ the respective shares of capital, ecosystem services, and labour in production. Defining the composite input $X \equiv E^{\beta/(1-\alpha)} H^{1-\beta/(1-\alpha)}$, we can write production function (8) as a Cobb-Douglas function with K and X as inputs:

$$F^{CD}(K, X) = AK^\alpha X^{1-\alpha}, \quad (9)$$

where X grows at a constant exogenous rate g , the weighted average of the growth rates of ecosystem services and effective labour input:

$$g = \left(1 - \frac{\beta}{1-\alpha}\right)g_H + \frac{\beta}{1-\alpha}g_E. \quad (10)$$

It follows immediately that on a balanced growth path, output F^{CD} and capital K grow at rate g . Brock and Taylor (2010) have a comparable approach but they consider the growth rate of the abatement technology in a problem with environmental pollution. Moreover, since the problem is now isomorphic to the standard Ramsey growth model with a Cobb-Douglas production function and an exogenous constant rate of technical change (where our composite input X replaces the effective labour input in the standard model, e.g. Acemoglu (2009)), we can state the following proposition, with a proof in Appendix A.

Proposition 1. For the Cobb-Douglas production function (8), the economy converges to a balanced growth path along which output, consumption and capital grow at growth rate g , given by (10), and the discount rate is constant at $r = \rho + \gamma g$. If $K_0 < (>) X_0(\alpha A/(\rho + \gamma g))^{1/(1-\alpha)}$, the discount rate de(in)creases monotonically over time along the transition towards balanced growth.

Equation (10) for the growth rate g has an easy interpretation. If the ecosystem services grow at the same rate as technology g_H , the economy grows at that rate as well, but if the ecosystem services grow at a lower or even negative rate g_E , the economy grows at a lower rate g . This downsizing effect depends on the share β of ecosystem services in the production. We do not have strong empirical evidence on the value of β , and this value will differ across sectors. In our simulations, we take a value of 0.2. We calibrate the model such that it generates, for a conventional value of 0.3 for the share of capital α , a value for the balanced growth rate g that is slightly below 2% (cf. Jones, 2016). We normalize the initial values of A , K_0 and X_0 , and choose conventional values for utility parameters ρ and γ (cf. Nordhaus, 2008; Stern, 2006). We choose a positive growth rate of ecosystem services close to 0. Investments in ecosystems can improve ecosystem services but this possibility is very limited. Our results can also be easily adjusted for a lower, possibly zero or negative, growth rate g_E of ecosystem services.

With the following parameter values:

$$\gamma = 1.45, g_H = 0.02, \rho = g_E = 0.001, \alpha = 0.3, \beta = 0.2, A = 0.1, K_0 = X_0 = 1, \quad (11)$$

the discount rate, which equals the marginal product of capital $\alpha A(K/X)^{\alpha-1}$, is initially equal to 3%, and converges to 2.2%, according to Proposition 1. The stable manifold of the system consisting of (A.2) and the second part of (A.1) yields the path for the discount rate in Fig. 1.

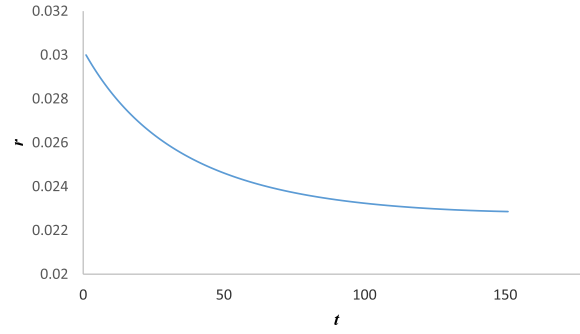


Fig. 1. Time path for discount rate: the Cobb-Douglas case

2.2. CES production

The Cobb-Douglas production function of the previous sub-section implies that the production elasticities of each of the inputs are constants, i.e. no matter how scarce or abundant an input is, a 1% increase has always the same proportionate effect on output, because the elasticity of substitution is one. In the current sub-section we generalize the production function to a CES specification in which production elasticities rise or fall with abundance, depending on whether the elasticity of substitution is above or below unity. We specify the CES production function as

$$F(K, E, H) = A \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta E^{\frac{\sigma-1}{\sigma}} + (1 - \alpha - \beta) H^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (12)$$

where A denotes total factor productivity and σ the elasticity of substitution; α and β are scale parameters and ensure that (12) converges to (8) for $\sigma = 1$. Aggregating the exogenously growing inputs into a composite input, which we again denote by X , we can write the production function as a CES function of the endogenously evolving state variable K and the exogenously evolving input variable X :

$$F(K, X) = A \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) X^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (13)$$

$$X(E, H) = \left(\frac{\beta}{1 - \alpha} E^{\frac{\sigma-1}{\sigma}} + \left(1 - \frac{\beta}{1 - \alpha} \right) H^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

From the second part of (13), it is easy to see that the growth rate g_X of the composite input X is a time-dependent weighted average of g_E and g_H , given by

$$g_X(t) = \omega(t)g_E + (1 - \omega(t))g_H, \quad (14)$$

$$\omega(t) = \frac{\beta E(t)^{\frac{\sigma-1}{\sigma}}}{\beta E(t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha - \beta) H(t)^{\frac{\sigma-1}{\sigma}}} = \frac{1}{1 + \left[\frac{1 - \alpha - \beta}{\beta} \left(\frac{H_0}{E_0} \right)^{\frac{\sigma-1}{\sigma}} \right] e^{\frac{\sigma-1}{\sigma} (g_H - g_E) t}}, \quad (15)$$

where the second equality in (15) uses the time paths of E and H from (7). Note that ω is a production elasticity: it is the partial elasticity of production inputs X with respect to ecosystem system services; it rises with ecosystem services scarcity if $\sigma < 1$.

We have assumed that $g_E < g_H$, so that ecosystem services become relatively scarcer over time. Then (15) implies that their weight in input growth ω is monotonically increasing over time towards 1, if $\sigma < 1$, and monotonically decreasing over time towards 0, if $\sigma > 1$. It follows from (14) that the growth rate of the composite production input X of ecosystem services and labour-augmenting technology, g_X , converges either to the growth rate of the ecosystem services g_E (if $\sigma < 1$) or to the rate of technological change g_H (if $\sigma > 1$). If man-made inputs can easily substitute for ecosystem services, the importance of the ecosystem services in production declines and technological change drives the economy in the end. However, if man-made inputs cannot easily substitute for ecosystem services, the low growth rate of ecosystem services restricts the growth

possibilities of the economy. The path of the growth rate g_X follows an inverse logistic growth curve. It starts between g_E and g_H , and moves down towards g_E or up towards g_H , depending on the elasticity of substitution σ .

From (14) and (15) it follows that the initial condition $g_X(0)$ is determined by initial conditions E_0 and H_0 . If $\sigma < 1$, so that the growth rate g_X converges to g_E in the long run, the initial condition $g_X(0)$ is close to g_H , provided that $\omega(0)$ is close to 0, which according to (15) requires $E_0 \gg H_0$. This means that if ecosystem services are initially abundant, the growth rate g_X starts close to g_H and it only comes down in order to converge to g_E when the ecosystem services become scarce. If $\sigma > 1$, so that the growth rate g_X converges to g_H in the long run, ecosystem services do not become scarce because of substitution.

In the long run the economy will converge to a balanced-growth path with the growth rate of the economy equal to g_E or to g_H . We characterize the steady state for system (7) in Proposition 2, with a proof in Appendix B.

Proposition 2. With CES production function (12), the economy converges to the steady state in which output, consumption, and capital grow at a common growth rate g^* and the discount rate converges to the steady-state value that is given by the Ramsey rule $r = \rho + \gamma g^*$. If the elasticity of substitution $\sigma < 1$, the steady-state growth rate of the economy, g^* , equals the growth rate of the ecosystem services g_E . If the elasticity of substitution $\sigma > 1$, the steady-state growth rate of the economy, g^* , equals the growth rate of the technological change g_H .

To analyze the dynamics, we write the system (7) in terms of two ratios $u = F/K$ and $v = C/K$ (see Appendix B). While F, K , and C keep growing in the steady state, these ratios are constant in the long run, as shown in Proposition 2. Here u denotes output per unit of capital, or capital productivity for short, and v denotes consumption per unit of capital, or consumption intensity for short. Because capital productivity and marginal productivity of capital (i.e. discount rate) are positively related, we can interpret u also as a monotone transformation of the discount rate r . In particular, from (13) we find

$$r = F_K = \alpha A (F/AK)^{\frac{1}{\sigma}} = \alpha A^{1-\frac{1}{\sigma}} u^{\frac{1}{\sigma}}. \quad (16)$$

In Fig. 2, we present the phase diagram in the (u, v) -plane. Since output F depends on the predetermined inputs K and X , the ratio $u = F/K$, and therefore also r , is a predetermined variable. As usual, $v = C/K$ is a non-predetermined variable. Different from the traditional Ramsey problem, as well as from the Cobb-Douglas case above, is that the input X develops with a non-constant growth rate. The growth rate g_X lies between the lower boundary g_E and the upper boundary g_H . The time-dependency of g_X causes one of the isoclines in the phase diagram to shift over time. In particular, u is constant if output and inputs grow at the same rate, which requires that $\dot{K}/K = u - v = g_X$. Accordingly, the extreme positions of the $\dot{u} = 0$ isocline are the black line given by $v = u - g_E$ and the grey line given by $v = u - g_H$.

The $\dot{v} = 0$ isocline is given by $v = u - \gamma^{-1}(F_K(u) - \rho)$. It is the black curve that cuts the line $v = u - g_E$ in (u_E, v_E) , with $u_E = F_K^{-1}(\rho + \gamma g_E)$, and the line $v = u - g_H$ in (u_H, v_H) , with $u_H = F_K^{-1}(\rho + \gamma g_H)$. This isocline slopes upwards in our case with $\gamma > 1$: constant v requires that consumption and capital grow at the same rate, which requires that faster capital accumulation associated with a higher capital productivity u is offset by lower savings associated with a larger consumption intensity v .

In case the growth rate g_X was fixed at either g_E or g_H , we would find the standard stable manifolds (black and grey broken curves in Fig. 2) through (u_E, v_E) and (u_H, v_H) , respectively. The left panel in Fig. 2 has an initial condition $u(0) > u_H$, and the right panel in Fig. 2 has an initial condition $u_E < u(0) < u_H$. The red curves indicate the stable manifolds for these two initial conditions.

First, we focus on the left panel of Fig. 2. It is helpful to make the following thought experiment. Suppose that the growth rate starts at g_H but suddenly and unexpectedly drops to g_E . It follows that the economy before the shock proceeds along the stable manifold that is approaching (u_H, v_H) but when the shock occurs, the economy jumps to the stable manifold that is approaching (u_E, v_E) . In our case, we do not have an unexpected shock so that the economy will prepare for a decline in the growth rate. As we have seen above in (14) and (15): if the elasticity of substitution is small, $\sigma < 1$, and ecosystem services are initially abundant, $E_0 \gg H_0$, the growth rate g_X starts close to g_H and stays close for some time, but then declines and

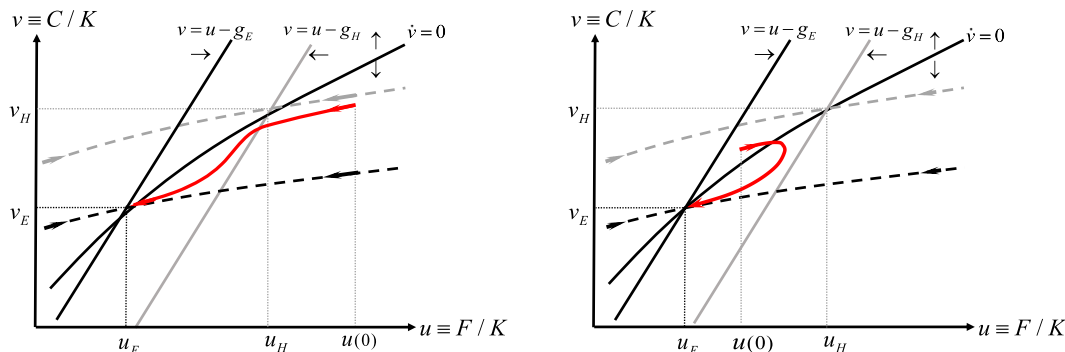


Fig. 2. Phase diagrams for high and low $u(0)$, respectively; $\sigma < 1 < \gamma$, $g_E < g_H$

ultimately converges to g_E . In the left panel of Fig. 2, this implies that the stable manifold of the system in u and v starts close to the upper stable manifold in the figure, but it needs to come down and converge to the lower stable manifold in the figure. In any case, the stable manifold lies between those upper and the lower stable manifolds.

In order to derive results about the time pattern of the discount rate, we take a closer look at the phase diagrams in Fig. 2. We consider the case with $\sigma < 1$, in which the economy converges in the long run to the balanced-growth steady state (u_E, v_E) . For any point in time t , we can draw the $\dot{u} = 0$ isocline in the phase diagrams in Fig. 2 as the line $v = u - g_X(t)$. This line lies parallel to, and in between, the two straight lines in Fig. 2 that represent $v = u - g_E$ and $v = u - g_H$. The line $v = u - g_X(t)$ is moving to the left over time, because g_X declines over time. It starts close to the line $v = u - g_H$, and it converges to the line $v = u - g_E$. To the right of the line $v = u - g_X(t)$, u is decreasing and to the left of this line, u is increasing. If the stable manifold of the system starts to the right of this line, it cannot cross this line, because u has to move down. This happens for $u(0) \geq u_H$, as in the left panel of Fig. 2. This also happens for $u(0) < u_H$ in case the growth rate g_X starts low, so that the line $v = u - g_X(t)$ has already moved sufficiently to the left and the stable manifold starts to the right of this line.

However, for $u(0) < u_H$, and in case the growth rate $g_X(t)$ starts close to g_H , the right panel of Fig. 2 applies. As long as the line $v = u - g_X(t)$ is only slowly moving to the left, the stable manifold lies close to the upper stable manifold (the grey broken curve), and it has a positive slope implying that both capital productivity, u , and consumption intensity, v , initially increase. However, since the economy ultimately has to converge to the balanced-growth steady state (u_E, v_E) , the stable manifold has to cross the $\dot{v} = 0$ isocline, so that at some point in time v starts moving down. Furthermore, it has to meet and cross the line $v = u - g_X(t)$ (that is moving to the left), so that u at some later point in time starts moving down as well. In order to show the conditions for these critical points, we note that the slope of the stable manifold for the system (B.3) in Appendix B is given by $dv/du = \dot{v}/\dot{u}$, where $\text{sign}[\dot{u}] = \text{sign}[g_X(t) + v - u]$. Accordingly, when the stable manifold crosses the $\dot{v} = 0$ isocline, the slope is 0. Furthermore, when it crosses the line $v = u - g_X(t)$, the slope is infinite, since then $\dot{u} = 0$. Therefore, when the stable manifold crosses the line $v = u - g_X(t)$, the curve changes direction, which implies that u starts decreasing after having increased first (see the right panel in Fig. 2).

Repeating the analysis for the case with $\sigma > 1$, we conclude that in case of good substitution the economy converges in the long run to the balanced-growth steady state (u_H, v_H) . The analysis is basically the same, but a mirror of the analysis above.

Equation (16) shows that the discount rate and the capital productivity u are positively related. Thus, the phase diagrams above directly inform us about the time patterns of the discount rate. We can derive an important qualitative conclusion from the phase diagram in the left panel of Fig. 2. If the economy starts with sufficiently abundant inputs X relative to capital K , so that $u(0) \geq u_H$, and if in addition the possibility of substitution of ecosystem services is poor, i.e. $\sigma < 1$, then the discount rate declines monotonically over time, i.e. the term structure is declining.

In order to find the precise time path of the discount rate $r = F_K(u)$, we have to solve the system (7), or equivalently the system (B.3) in Appendix B, in order to find the time path of u . It is not possible to solve this system analytically, so that we have to resort to numerical methods. The time paths of the growth rate g_X and the exogenous inputs H and E are given by (14), (15) and (7), with the initial conditions E_0 and H_0 . Our algorithm searches a time T , where the growth rate g_X has converged close to the steady state g^* . Then the algorithm calculates $H(T)$ and $E(T)$, and uses the steady-state values u^* and v^* from (B.6) and (16) to calculate approximations of $K(T)$, $C(T)$ and $F(T)$ with (12) and with the definitions $u = F/K$ and $v = C/K$. With these final values, a standard algorithm for the Ramsey growth model yields the time paths for K and F , and thus for u , and thus for the discount rate. We could also apply some algorithm directly to the two-point boundary value problem (B.3) in Appendix B, but that would amount to the same thing.

For $\sigma < 1$, g^* is equal to g_E . This is the case of limited substitutability of ecosystem services in production. This case is relevant, if natural resources and man-made inputs are poor substitutes. It is not yet clear from the empirical literature how important this is. In a paper on substitutability of energy in production, van der Werf (2008) estimates the substitution elasticities to lie between 0.17 and 0.65. On the other hand, in a paper on substitutability of natural capital, Markandya and Pedroso-Galinato (2007) conclude that the elasticities of substitution are larger than 1. More research on these empirical aspects is needed. However, this paper provides the analytical framework for analysing the discount rate in case of limited substitutability of ecosystem services in production.

We take the same parameter values as in (11). In order to calibrate the model to the stylized fact that the average (structural) growth rate has been trendless over the past decades (e.g., Jones, 2016), we take the initial values for E and H such that $X_0 = 1$, according to (13), and $g_X(0) = 0.99g_H$, according to (14) and (15). Together with $K_0 = 1$ it follows that $u(0) = F(K_0, X_0)/K_0 = A = 0.1$, so that the discount rate $F_K(u)$ is again initially equal to 3% (for all σ) according to (16), and converges to 0.245% according to the Ramsey rule. In the phase diagram in Fig. 2, it means that we start in $u(0) = u_H$ and that $v(0)$ jumps to the stable manifold close to (u_H, v_H) , because the initial growth rate $g_X(0)$ is close to g_H . As long as the growth rate g_X stays close to g_H , the optimal path hardly moves but when the growth rate g_X starts to decline, the line $v = u - g_X(t)$ moves to the left and the optimal path moves to the left and comes down, following the stable manifold. The growth rate g_X of the composite input X converges from the initial 0.0198 to the steady-state value, which is equal to g_E (i.e., 0.001). Consequently, the discount rate follows the same pattern, starting at 3%, staying just below 3% for some time, and converging to 0.245% in the long run.

Fig. 3 and Fig. 4 show the graphs for the growth rate g_X and the discount rate $r = F_K(u)$, for different values of the elasticity of substitution $\sigma < 1$. For comparison, we also show the growth rate and the discount rate for a value of $\sigma > 1$. For this value, the growth rate g_X of the composite input X converges to g_H (i.e., 0.02), according to (14) and (15), and the discount rate converges to 3%, according to the Ramsey rule. Because we start with $g_X(0)$ close to g_H , the growth rate remains high, and the

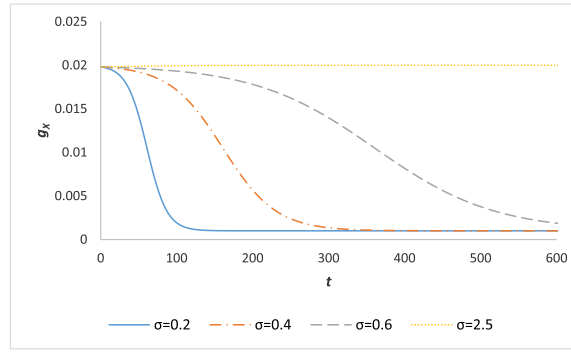


Fig. 3. Time paths for growth rate: the CES case

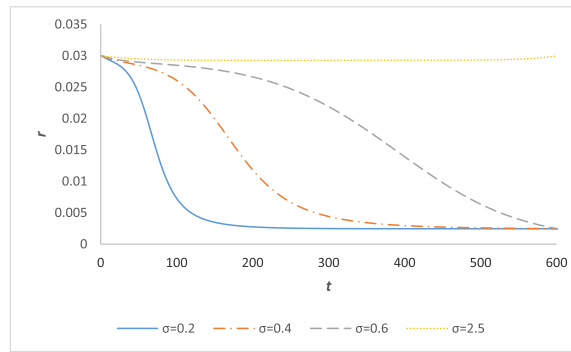


Fig. 4. Time paths for discount rate: the CES case

discount rate is almost constant. If man-made inputs can easily substitute for ecosystem services, the standard constant discount rate is the right choice in a deterministic setting, but if man-made inputs cannot easily substitute for ecosystem services, it is more appropriate to choose a declining discount rate.

The patterns in Figs. 3 and 4 are interesting and intuitively clear. If the growth rate and the discount rate decrease towards their low steady-state value, they decrease faster in case the elasticity of substitution σ is small. In that case, the effect of the low growth rate of the ecosystem services is stronger and drives down the growth rate of the economy and the discount rate faster. For policy, this implies that if the substitution possibilities are very limited, one can start with a discount rate of 3% but this number decreases relatively fast in the long term. If the substitution possibilities are better, but with the elasticity of substitution σ still smaller than 1, the decrease of the discount rate in the long term is slower.

An important conclusion is that we can get a declining discount rate. Uncertainty is the usual argument for the declining discount rate (Gollier, 2002, 2010, 2013; Newell and Pizer, 2003; Weitzman, 2007, 2010), but in this analysis it is the effect of the low growth rate of ecosystem services that are used in production and cannot easily be substituted.

3. Ecosystem services in utility and production

In this section, we take $\pi > 0$ in the utility function (5) in order to consider ecosystem services in both the utility function and production function. The Keynes-Ramsey rule for the optimal consumption path (6) becomes

$$\dot{C}(t) = \gamma_{CC}^{-1} (F_K(K(t), E(t), H(t)) - \gamma_{CE} g_E - \rho) C(t), \quad (17)$$

where $\gamma_{CC} = -CU_{CC}/U_C$ and $\gamma_{CE} = -EU_{CE}/U_C$ denote the elasticities of the marginal utility of consumption. Note that (17) implies that the discount rate can be written as

$$F_K = \rho + \gamma_{CC} g_C + \gamma_{CE} g_E, \quad (18)$$

where g_C denotes the growth rate of consumption (see Weikard and Zhu, 2005; Hoel and Sterner, 2007; Traeger, 2011). The two elasticities γ_{CC} and γ_{CE} are derived from (5) and can be written as:

$$\gamma_{CC} = \gamma - \gamma_{CE}, \gamma_{CE} = \delta(\gamma - \zeta^{-1}), \quad (19)$$

where

$$\delta = \frac{\pi E^{\frac{\zeta-1}{\zeta}}}{(1-\pi)C^{\frac{\zeta-1}{\zeta}} + \pi E^{\frac{\zeta-1}{\zeta}}} = \frac{\pi}{(1-\pi)(C/E)^{\frac{\zeta-1}{\zeta}} + \pi}. \quad (20)$$

This δ can be interpreted as the value share of the ecosystem services in the consumer expenditure. It is generally not constant over time if there is non-unitary substitutability between consumption and ecosystem services in utility. In [Hoel and Sterner \(2007\)](#) and [Traeger \(2011\)](#) the growth rates of C and E are fixed, so that δ converges to 0 or 1, depending on the elasticity of substitution ζ . However, if the growth rate of C converges to the growth rate of E , then δ converges to a number between 0 and 1.

In order to understand the results below, we first focus on the environmentally extended Ramsey rule for this problem, given by (18). Using (19), we can write this as:

$$F_K(t) = \rho + \gamma[(1-\delta(t))g_C(t) + \delta(t)g_E] + \zeta^{-1}\delta(t)[g_C(t) - g_E], \quad (21)$$

where δ is given by (20). Equation (21) clearly shows the determinants of the discount rate F_K . [Traeger \(2011\)](#) has the same result and labels the second and third term at the right-hand side of (21) as the “overall growth effect” and the “real substitutability effect”. Since we also have a second substitutability effect in the growth rate of consumption g_C , we prefer to label the third term as the “relative price effect”. It is the generalization of what [Hoel and Sterner \(2007\)](#) label as the relative price effect, where they consider the case in which δ has converged to 1. Note that the welfare at a point in time t not only depends on produced consumption C but on “full consumption”, i.e. C and ecosystem services E , weighted by the value shares $(1-\delta)$ and δ , respectively. The interpretation of (21) largely follows [Traeger \(2011\)](#). The second term shows that if full consumption grows fast, the discount rate becomes high, because the marginal value of full consumption falls. This is the standard story. The strength of this effect is governed by the elasticity of intertemporal substitution γ^{-1} (i.e., the intergenerational inequality aversion γ). The third term shows that if the consumption C becomes more abundant than the ecosystem services E , the relative price of consumption C becomes lower, provided it is an imperfect substitute for ecosystem services E . The consumption discount rate becomes higher. Therefore, the growing relative scarcity of ecosystem services provides an additional reason to value future consumption lower, and thus to use a higher consumption discount rate. The strength of this effect is governed by the elasticity of intra-temporal substitution ζ .

For a given growth rate of consumption g_C , a decline in the growth rate of ecosystem services g_E has two opposing effects. On the one hand, it slows down full consumption growth, and thus lowers the discount rate. On the other hand, it makes consumption C relatively more abundant, and therefore increases the (consumption) discount rate. The second effect dominates, if the elasticity of intra-temporal substitution ζ is smaller than the elasticity of intertemporal substitution γ^{-1} .

[Traeger \(2011\)](#) writes equation (21) as $F_K(t) = \rho + \gamma g_C - (\gamma - \zeta^{-1})\delta(t)[g_C - g_E]$. This equation shows that even for a fixed growth rate of consumption g_C , the discount rate is not constant: the time-varying value share of ecosystem services δ makes the discount rate time-dependent. The discount rate is declining in the case $\zeta > 1$, so that δ converges to 0, with $\gamma < \zeta^{-1}$, and in the case $\zeta < 1$, so that δ converges to 1, with $\gamma > \zeta^{-1}$. In the sequel, we will incorporate this decline in our analysis, focussing on the second case because these parameter values are more interesting.

As in the previous section, we assume that the economy is driven by labour-augmenting technological change H , with growth rate g_H , but that the ecosystem services, which provide intermediate inputs in production, grow at a lower rate g_E . Instead of (7), we have to solve the system

$$\begin{aligned} \dot{K}(t) &= F(K(t), E(t), H(t)) - C(t), K(0) = K_0, \\ \dot{C}(t) &= (\gamma - \gamma_{CE}(t))^{-1}(F_K(K(t), E(t), H(t)) - \gamma_{CE}(t)g_E - \rho)C(t), \\ \dot{H}(t) &= g_H H(t), H(0) = H_0, \\ \dot{E}(t) &= g_E E(t), E(0) = E_0, \end{aligned} \quad (22)$$

where γ_{CE} is given by (19) and (20). Note that γ_{CE} is function of $C(t)$ and $E(t)$.

In this system, some ecosystem services E provide an amenity value to consumers, and therefore affect the elasticities of the marginal utility of consumption. Other ecosystem services E are a production factor, and thus affect the growth rate of consumption. It follows that the ecosystem services E affect the discount rate in two ways.

In order to investigate the long-run properties of the system, we can follow the analysis of the previous section. In the right-hand side of the second equation of system (B.3) in [Appendix B](#), however, the growth rate of consumption in equation (6) changes into the growth rate of consumption in equation (17). Furthermore, γ_{CE} in equation (22) can be replaced by $\delta(\gamma - \zeta^{-1})$, according to (19). This leads to the more complicated system

$$\begin{aligned}
\dot{u}(t) &= [(1 - F_K(u(t)))/u(t)](g_X(t) + v(t) - u(t))u(t), \\
\dot{v}(t) &= \left[\left(\gamma - \delta(t) \left(\gamma - \zeta^{-1} \right) \right)^{-1} \left(F_K(u(t)) - \delta(t) \left(\gamma - \zeta^{-1} \right) g_E - \rho \right) \right] v(t), \\
g_X(t) &= \omega(t)g_E + (1 - \omega(t))g_H,
\end{aligned} \tag{23}$$

where (15) gives the time path $\omega(t)$ and δ is given by (20), which is a function of C/E . It is tedious, but straightforward, to express the variable C/E in the variables u , v and g_X (see Appendix C), so that (23) is a well-defined system.

As in the previous section, the $\dot{u} = 0$ isocline is the line $v = u - g_X(t)$, which is located in between the lines $v = u - g_E$ and $v = u - g_H$ in the (u, v) -plane. We can characterize the $\dot{v} = 0$ isocline by two extreme positions as well. For $\delta = 0$, the $\dot{v} = 0$ isocline is the same as in the previous section, and for $\delta = 1$, the $\dot{v} = 0$ isocline reaches the other extreme (because $0 \leq \delta \leq 1$):

$$\begin{aligned}
v &= u - \gamma^{-1}(F_K(u) - \rho), \quad \delta = 0, \\
v &= u - \zeta \left(F_K(u) - \left(\gamma - \zeta^{-1} \right) g_E - \rho \right), \quad \delta = 1.
\end{aligned} \tag{24}$$

Note that the two curves in (24) coincide for $\gamma = \zeta^{-1}$, and that both curves in (24) cut the line $v = u - g_E$ in (u_E, v_E) , with $u_E = F_K^{-1}(\rho + \gamma g_E)$. For $\gamma > \zeta^{-1}$, the second curve in (24) cuts the line $v = u - g_H$ in a point with $u < u_H$, and for $\gamma < \zeta^{-1}$, the second curve in (24) cuts the line $v = u - g_H$ in a point with $u > u_H$, where $u_H = F_K^{-1}(\rho + \gamma g_H)$.

The literature shows a wide range of estimates for the elasticity of substitution between consumption and ecosystem services in utility ζ (Drupp, 2018). The literature is also not conclusive about the value of the elasticity of intertemporal substitution γ^{-1} , but most of the literature uses values for γ around 1.5. We cannot rule out that $\gamma < \zeta^{-1}$, but the results in Drupp (2018) and conventional estimates for γ imply that it is reasonable to assume that $\gamma > \zeta^{-1}$. For this case, we can depict the extreme isoclines and the steady states of the phase diagram in the (u, v) -plane as in Fig. 5.

The steady-state version of the extended Ramsey rule becomes:

$$F_K(u^*) = \rho + \delta^* \left(\gamma - \zeta^{-1} \right) g_E + \left(\gamma - \delta^* \left(\gamma - \zeta^{-1} \right) \right) g^*, \tag{25}$$

where δ^* denotes the steady-state value of δ .

Depending on the elasticities of substitution ζ and σ , we can distinguish three cases that correspond to the three possible steady states in Fig. 5.

Case 1. If $\zeta > 1$ and $\sigma > 1$, g_X converges to g_H and δ converges to 0. The steady state is the intersection of the first curve in (24) and the line $v = u - g_H$, and the discount rate becomes

$$F_K(u^*) = \rho + \gamma g_H, \tag{26}$$

which is the case in which it is simply assumed that the economy can substitute away from ecosystem services.

Case 2. If $\zeta < 1$ and $\sigma > 1$, g_X converges to g_H and δ converges to 1. The steady state is the intersection of the second curve in (24) and the line $v = u - g_H$, and the discount rate becomes

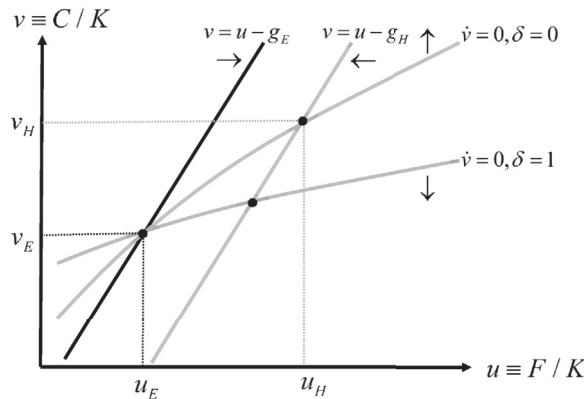


Fig. 5. Steady states for system (23) with $\gamma > \zeta^{-1}$

$$F_K(u^*) = \rho + (\gamma - \zeta^{-1})g_E + \zeta^{-1}g_H = \rho + \gamma g_H - (\gamma - \zeta^{-1})(g_H - g_E), \quad (27)$$

which is the case in which consumers cannot easily substitute produced consumption goods for the amenity value of ecosystem services, although the producers can easily substitute away from the intermediate use of ecosystem services in the production. As we have shown in Fig. 5, for $\gamma > \zeta^{-1}$, the second curve in (24) cuts the line $v = u - g_H$ in a point with $u^* < u_H$, where $u_H = F_K^{-1}(\rho + \gamma g_H)$, so that the steady-state discount rate is lower than in Case 1. For $\gamma < \zeta^{-1}$, the second curve in (24) cuts the line $v = u - g_H$ in a point with $u^* > u_H$, so that the steady-state discount rate is higher than in Case 1 (see also Hoel and Sterner, 2007).

Case 3. If $\sigma < 1$, g_X converges to g_E and δ converges to some number in between 0 and 1. This number is determined by ζ and ζ and by the steady-state values u^* and v^* (see Appendix C). The steady state is the intersection of the line $v = u - g_E$ and a $\dot{v} = 0$ isocline in between the two curves in (24). The steady state discount rate becomes

$$F_K(u^*) = \rho + \gamma g_E, \quad (28)$$

which is the case where the limited substitutability of ecosystem services in production restricts the economic growth. At the end, the substitutability in utility does not matter, because the growth rate of the economy in the long run converges to g_E .

In order to study the development of the discount rate over time, we consider the phase diagram in Fig. 5 more closely. For fixed values of δ between 0 and 1, we consider the $\dot{v} = 0$ isoclines given by

$$v = u - (\gamma - \delta(\gamma - \zeta^{-1}))^{-1} (F_K(u) - \delta(\gamma - \zeta^{-1})g_E - \rho), \quad (29)$$

which are positioned between the extremes for $\delta = 0$ and $\delta = 1$ in equation (24). In Fig. 6, we have depicted the $\dot{v} = 0$ isoclines for a fixed $\delta(0)$ and two different values of ζ . It is easy to show that the $\dot{v} = 0$ isocline in equation (29) rotates downwards for an increasing ζ .

Note that $\delta(0)$ in (20) is not predetermined, because $C(0)$ is not predetermined. It is reasonable to assume that initially the value share of ecosystem services in consumer expenditure is small, so that $\delta(0)$ is close to 0, and both corresponding $\dot{v} = 0$ isoclines in Fig. 6 are close to the one for $\delta = 0$. It is clear from (20) that δ is decreasing over time if $\zeta > 1$ (where ζ is the elasticity of substitution in utility), and that δ is increasing over time if $\zeta < 1$. It implies that not only the line $v = u - g_X(t)$ is moving to the right or to the left in the dynamical process, but that also the $\dot{v} = 0$ isocline is rotating upwards (in case $\zeta > 1$) or downwards (in case $\zeta < 1$).

In cases 1 and 2, the line $v = u - g_X(t)$ starts close to the line $v = u - g_H$, and moves towards this line. If we start in the initial condition $u(0) = u_H = 0.1$, with the discount rate equal to 3%, we cannot immediately jump close to the point (u_H, v_H) , as in the previous section, because we have to stay below the $\dot{v} = 0$ isoclines. This implies that the optimal path moves a bit to the left until it is blocked by the line $v = u - g_X(t)$, so that the discount rate initially decreases a bit. If $\zeta > 1$, δ converges to 0 and the $\dot{v} = 0$ isocline rotates upwards, so that the optimal path changes direction, and the discount rate converges to 3%. Easy substitution of ecosystem services in utility implies that the path of the discount rate is close to the one we found in the previous section. However, if $\zeta < 1$, then δ converges to 1 and the $\dot{v} = 0$ isocline rotates downwards, so that the optimal path moves down and to the left, and the discount rate declines in the long run to the steady-state value given by (27). This shows the relative price effect as was introduced in the previous literature (Weikard and Zhu, 2005; Hoel and Sterner, 2007).

In case 3, the line $v = u - g_X(t)$ starts close to the line $v = u - g_H$ and moves to the left. The $\dot{v} = 0$ isocline rotates upwards (in case $\zeta > 1$) or downwards (in case $\zeta < 1$), but not all the way to the extreme positions in (24). The value share of ecosystem services in consumer expenditure δ converges to a long-run value in between $\delta(0)$ and 0 (in case $\zeta > 1$), or to a long-run value

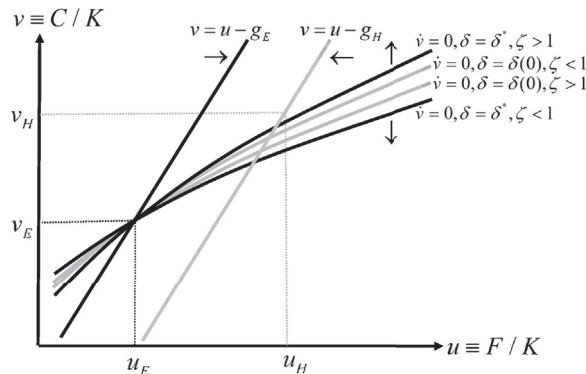


Fig. 6. Dynamics of $\dot{v} = 0$ isoclines for $\gamma^{-1} < \zeta_1 < 1$ and $\zeta_2 > 1$.

in between $\delta(0)$ and 1 (in case $\zeta < 1$). These values can be calculated with the expressions in Appendix C. The main difference with cases 1 and 2 above is that the line $v = u - g_X(t)$ moves to the left, so that the optimal path will come down at some point and converge to the steady state (u_E, v_E) . This implies that the relative price effect almost disappears in the long run, because the growth rate of consumption converges to the growth rate of the ecosystem services. In the short run, however, the growth rate of consumption is higher than the growth rate of ecosystem services, and a similar effect occurs as in cases 1 and 2. If we start in $u(0) = u_H = 0.1$, with the discount rate equal to 3%, we have to stay below the $\dot{v} = 0$ isoclines. This implies that the optimal path starts to move a bit to the left, until it is temporarily blocked by the line $v = u - g_X(t)$, so that the discount rate initially decreases a bit. This effect is small, but it is a little bit larger in case $\zeta > 1$ than in case $\zeta < 1$, because the initial position of the $\dot{v} = 0$ isocline is a bit lower in the first case. Otherwise, the optimal paths are not much different from the one described in the previous section. The production effect leading to a declining discount rate dominates the relative price effect in utility. In the next section, we will present an example.

We summarize our findings from the phase diagram in Proposition 3.

Proposition 3. For the CES production function (12) and CES utility function (5), the economy converges to the steady state in which output, consumption and capital grow at a common growth rate g^* . If the elasticity of substitution in production is small, $\sigma < 1$, the steady-state growth rate of the economy, g^* , equals the growth rate of the ecosystem services, g_E , and the discount rate converges to the steady-state value given by the standard Ramsey rule $r = \rho + \gamma g_E$. However, if the elasticity of substitution in production is large, $\sigma > 1$, the steady-state growth rate of the economy, g^* , equals the growth rate of technology, g_H . Then the long-run discount rate takes one of two forms: (i) if the elasticity of substitution in utility is large, $\zeta > 1$, it converges to the steady-state value given by the standard Ramsey rule $r = \rho + \gamma g_H$; (ii) if the elasticity of substitution in utility is small, $\zeta < 1$, it converges to the steady-state value given by an adjusted Ramsey rule $r = \rho + \gamma g_H - (\gamma - \zeta^{-1})(g_H - g_E)$. If the economy starts with sufficiently abundant inputs X relative to capital K , so that $u(0) \geq u_H$, and if $\sigma < 1$ and $\gamma > \zeta^{-1}$, then the discount rate declines monotonically over time, i.e. the term structure is declining.

4. Numerical example

In order to present the precise time path of the discount rate in the three cases that we have identified in the previous section, we have to resort to numerical methods again. We use the same algorithm as in section 2. For each case, we again take the parameter values and initial values given in (11). We choose the share π in the utility function (5) such that the initial value share of the ecosystem services in the consumer expenditure $\delta(0) = 0.2$. This requires an iterative application of our algorithm, because $\delta(0)$ is not predetermined. We take different values for ζ and σ in order to distinguish the three cases of the previous section. For σ , we return to the central values of Section 2.2: $\sigma = 2.5 > 1$ and $\sigma = 0.4 < 1$. For ζ , we take the mean value and the lowest value from the empirical literature (Drupp, 2018): $\zeta = 2.31 > 1$ and $\zeta = 0.86 < 1$, respectively. The resulting four cases are summarized and labelled in Table 1. In the phase diagram in Fig. 6, the $\dot{v} = 0$ isocline rotates upwards for $\zeta = 2.31$ and it rotates downwards for $\zeta = 0.86$.

Fig. 7 shows the time paths of the discount rate. Fig. 8 shows the time paths of the value share of ecosystem services in consumer expenditure δ . The discount rate starts at 3% in all cases. In case 1, the discount rate remains close to this level, and the time path is almost flat, because the growth rate is high and ecosystem services can be easily substituted in production and in utility. As we have seen in our discussion of the phase diagram in Fig. 6, initially the discount rate moves down a bit, but then it moves up again and converges to the steady-state value 3%. This is the case in which the ecosystem services are perceived to be substitutable in production and consumption.

In case 2, the growth rate also remains high, because ecosystem services can be easily substituted in production. In utility, however, the ecosystem services become relatively scarce and since we assume on the basis of empirical observations that the elasticity of intra-temporal substitution ζ is larger than the elasticity of intertemporal substitution γ^{-1} , the discount rate decreases further. It is interesting to note that it takes a long time before the decrease becomes substantial. Within the 350 times period shown in the figure, the discount rate is still far from its steady-state value, which is $F_K(u^*) = 2.5\%$ according to (27). This makes clear that the relative price effect is small, and kicks in only after a long time.

In case 3, the discount rate strongly declines because the growth rate declines. Fig. 7 shows that cases 3a and 3b hardly differ. In the beginning, the paths are a little bit different. As we have seen in our discussion of the phase diagram in Fig. 6, initially the discount rate moves down a bit more in case 3a than in case 3b. Then the relative price effect kicks in. Fig. 8 shows how the value share of ecosystem services δ in consumer expenditure develops in these two cases, with different steady-state values δ^* (i.e., $\delta^* = 0.034$ in case 3a and $\delta^* = 0.304$ in case 3b). This causes the path of the discount rate in case 3b to move

Table 1
 π and δ^* in four cases; $\delta(0) = 0.2$

	$\zeta = 2.31 > 1$	$\zeta = 0.86 < 1$
$\sigma = 2.5 > 1$	case 1 $\pi = 0.563, \delta^* = 0$	case 2 $\pi = 0.133, \delta^* = 1$
$\sigma = 0.4 < 1$	case 3a $\pi = 0.0166, \delta^* = 0.034$	case 3b $\pi = 0.351, \delta^* = 0.304$

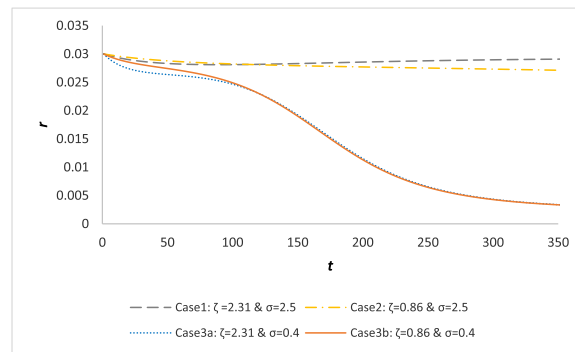


Fig. 7. Time paths for the discount rate.

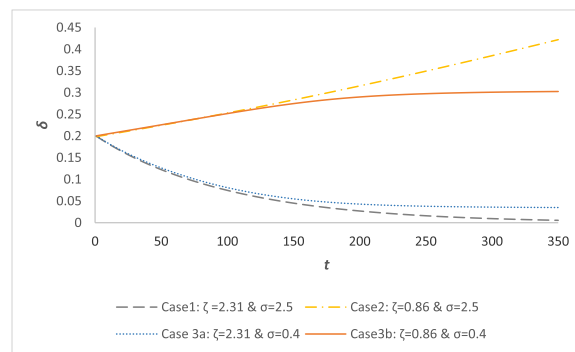


Fig. 8. Time paths for the value share of ecosystem services.

down slowly and to get closer to the path in case 3a. Fig. 7 shows that when the two paths for the discount rate meet in these cases, the effect of the scarcity of ecosystem services in production kicks in. This dominates the relative price effect in utility, and the two paths come down and stay close together. In the long run, the discount rate converges in both cases to the steady-state value 0.245%, according to (28).

5. Conclusion

This paper considers the discount rate in case ecosystem services are important for production but cannot grow at the same rate as the usual drivers of economic growth, such as technological change. The literature on the discount rate mostly assumes that the growth rate is given but in case ecosystem services are important for production and cannot be easily substituted, growth of the economy will be restricted and the discount rate will decline.

This paper considers the ecosystem services in production and directly in utility, and allows for different elasticities of substitution. If substitution is easy, the discount rate in the long term follows the standard Ramsey rule with a high growth rate. On the balanced-growth path, discounting over any time horizon occurs at a constant rate. If substitution in production is easy but substitution in utility is not easy, the relative price effect from the previous literature shows up; growing scarcity of ecosystem services then requires an adjusted Ramsey rule for the discount rate. More importantly, however, if substitution in production is not easy, the discount rate declines towards a low value given by the standard Ramsey rule with a low growth rate. Moreover, in this context the relative price effect is small and almost disappears in the long run.

In order to analyse these issues, this paper uses a Ramsey growth model, with both a CES utility function and a CES production function. The balanced-growth steady states are derived, the system dynamics are characterized in phase diagrams, and a numerical procedure is used to calculate the paths of the growth rate and the discount rate in a calibrated model. In this way, the paper compares, across different scenarios, not only the steady-state values of the discount rate but also the paths towards these steady-state values.

The main result of the paper is that the appropriate discounting rule crucially depends on the role of ecosystem services in production. If the ecosystem services can be easily substituted, growing scarcity will not slow down the growth from technological change, and the discount rate can be based on the current balanced-growth rate of consumption. However, if the ecosystem services cannot be easily substituted, and if the growth rate of ecosystem services is low, future growth will slow down and the discount rate will decline towards a low value. This main result implies that the role of ecosystem services

in utility is not very strong. In the case in which the economy can achieve a high growth rate in the long term, the previous literature has already pointed out the relative price effect that occurs if the growth rate of the ecosystem services and the elasticity of substitution in utility are low. However, we show that this effect is not large and only occurs after a long term. In the case in which the economy cannot achieve a high growth rate in the long term, we show that this relative price effect hardly plays a role. These are theoretical results. In practice, it is necessary to identify production sectors where ecosystem services are essential, and to determine the growth rates and the elasticities of substitution. This is a topic for further research.

The previous literature has shown extensively that a declining discount rate may also result from the introduction of several forms of uncertainty. This paper abstracts from uncertainty, but it is clear that many arguments point in the same direction, and thus support the idea of a declining discount rate. This paper has specifically focused on the role of a limited availability and substitutability of ecosystem services in production. A declining discount rate is very important for the cost-benefit analyses with a long time horizon, because a flat discount rate would make the costs and benefits in the far future hardly count.

This paper assumes an exogenous (low) growth rate for ecosystem services, but ignores a possible feedback effect of production and consumption (and thus emissions into the natural environment) on the availability of the ecosystem services (see also Brock and Xepapadeas, 2018). This paper contains a number of interesting simulations to illustrate the theoretical results, but calibrations with real data are needed in order to quantify the policy advice. These are also topics for future research.

Appendix A. Proof of the Proposition 1

It is convenient to define the variables $c = C/X$ and $k = K/X$ and to rewrite problem (1), using $X(t) = X_0 e^{gt}$, as follows:

$$\max_{c(t)} \int_0^{\infty} e^{-(\rho+(\gamma-1)g)t} X_0^{1-\gamma} U(c(t)) dt, U(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad (A.1)$$

$$\dot{k}(t) = f(k(t)) - gk(t) - c(t), f(k) = Ak^{\alpha}, k(0) = K_0/X_0.$$

The Keynes-Ramsey rule becomes

$$\dot{c}(t) = \gamma^{-1} (f'(k(t)) - \rho - \gamma g) c(t) \quad (A.2)$$

In the steady state (k^*, c^*) , consumption C , capital K and output F^{CD} grow at the same rate g , and the discount rate is constant and given by the Ramsey rule

$$f'(k^*) = \rho + \gamma g. \quad (A.3)$$

The specification of the production function f in (A.1) implies that $f'(k^*) = \alpha A(k^*)^{\alpha-1}$. It follows from (A.3) that $K(t)/X(t) = k^* = (\alpha A/(\rho + \gamma g))^{1/(1-\alpha)}$ along the balanced growth path. The transitional dynamics is monotonic. If $k(0) = K_0/X_0 < (>) k^*$, then $k(t)$ converges to k^* from below (above). It follows that the discount rate de(in)creases monotonically over time on the transition path. Q.E.D.

Appendix B. Proof of the Proposition 2

It is convenient to define the variables $u = F/K$ and $v = C/K$, and rewrite system (7). The variable u denotes output per unit of capital, and the variable v denotes consumption per unit of capital. The difference is investment per unit of capital, so that the capital accumulation in (7) can be written as:

$$\dot{K}(t) / K(t) = u(t) - v(t). \quad (B.1)$$

Differentiating F in (13) with respect to K we find F_K , which can be written as a function of capital productivity u as in (16). From the first part of (13), we find

$$\frac{\dot{F}(t)}{F(t)} = \varepsilon_K(t) \frac{\dot{K}(t)}{K(t)} + (1 - \varepsilon_K(t)) \frac{\dot{X}(t)}{X(t)} = \varepsilon_K(t) \frac{\dot{K}(t)}{K(t)} + (1 - \varepsilon_K(t)) g_X(t), \quad (B.2)$$

where the production elasticity ε_K of capital K is equal to $\varepsilon_K = KF_K/F = F_K/u$. It follows that $\dot{u}(t)/u(t) = \dot{F}(t)/F(t) - \dot{K}(t)/K(t) = (1 - \varepsilon_K(t))(g_X(t) - \dot{K}(t)/K(t))$. Note that $\dot{v}(t)/v(t) = \dot{C}(t)/C(t) - \dot{K}(t)/K(t)$. Using (B.1) and (14), it follows now that system (7) can also be written as a system in (u, v, g_X) :

$$\begin{aligned}\dot{u}(t) &= [(1 - F_K(u(t)))/u(t)](g_X(t) + v(t) - u(t))u(t), \\ \dot{v}(t) &= [\gamma^{-1}(F_K(u(t)) - \rho) + v(t) - u(t)]v(t), \\ g_X(t) &= \omega(t)g_E + (1 - \omega(t))g_H,\end{aligned}\tag{B.3}$$

where (15) gives the time path $\omega(t)$. According to (14) and (15), the growth rate g_X converges in the long run to:

$$\lim_{t \rightarrow \infty} g_X(t) = g^* = \begin{cases} g_E & \text{if } \sigma < 1 \\ g & \text{if } \sigma = 1 \\ g_H & \text{if } \sigma > 1 \end{cases}.\tag{B.4}$$

The initial condition $u(0)$ is equal to $F(K_0, E_0, H_0)/K_0$, and the initial condition of $g_X(0)$ is given by (14) and (15), but the initial condition of the consumption intensity $v(0)$ is not predetermined. The $\dot{u} = 0$ and $\dot{v} = 0$ isoclines are given by, respectively

$$v = u - g_X, v = u - \gamma^{-1}(F_K(u) - \rho)\tag{B.5}$$

It follows that the steady state of system (B.3) becomes (u^*, v^*, g^*) where

$$F_K(u^*) = \rho + \gamma g^*, v^* = u^* - g^*, g_X = g^*.\tag{B.6}$$

This means that the steady-state discount rate is given by the Ramsey rule. In the steady state (u^*, v^*, g^*) , output, consumption and capital grow at the same rate g^* , which is given by (B.4). The steady-state growth rate g^* is equal to the steady-state output per unit of capital rate u^* minus the steady-state consumption per unit of capital v^* , which amounts to steady-state investment per unit of capital. Q.E.D.

Appendix C. Specification of δ as a function of the variables of system (23)

According to (20), δ is a function of C/E .

Since $C/E = (C/K)(K/F)(F/E) = (v/u)(F/E)$, we have to show that F/E is a function of the system variables. From (12), it follows that:

$$F_E = \beta A^{\frac{\sigma-1}{\sigma}} \left(\frac{F}{E}\right)^{\frac{1}{\sigma}} \Rightarrow \frac{F}{E} = A \left(\frac{1}{\beta} \frac{EF_E}{F}\right)^{\frac{\sigma}{1-\sigma}},\tag{C.1}$$

and from (13) and (12), it follows that:

$$\frac{\dot{F}}{F} = \frac{KF_K}{F} \frac{\dot{K}}{K} + \left(1 - \frac{KF_K}{F}\right) \frac{\dot{X}}{X} = \frac{KF_K}{F} \frac{\dot{K}}{K} + \frac{EF_E}{F} \frac{\dot{E}}{E} + \left(1 - \frac{KF_K}{F} - \frac{EF_E}{F}\right) \frac{\dot{H}}{H}.\tag{C.2}$$

With (16) and the definitions of the growth rates, this implies that:

$$\left(1 - \frac{F_K(u)}{u}\right)g_X = \frac{EF_E}{F}g_E + \left(1 - \frac{F_K(u)}{u} - \frac{EF_E}{F}\right)g_H.\tag{C.3}$$

Combining (C.1) and (C.3) yields:

$$\frac{C}{E} = \frac{v}{u} \frac{F}{E} = \frac{Av}{u} \left(\frac{1}{\beta} \left(\frac{g_H - g_X}{g_H - g_E}\right) \left(1 - \frac{F_K(u)}{u}\right)\right)^{\frac{\sigma}{1-\sigma}}.\tag{C.4}$$

Q.E.D.

References

- Acemoglu, D., 2009. *Introduction to Modern Economic Growth*. Princeton University Press, Princeton NJ.
- Acemoglu, D., Guerrieri, V., 2008. Capital deepening and nonbalanced economic growth. *J. Political Econ.* 116, 467–498.
- Baumgärtner, S., Klein, A.M., Thiel, D., Winkler, K., 2015. Ramsey discounting of ecosystem services. *Environ. Resour. Econ.* 61, 273–296.
- Brock, W.A., Taylor, M.S., 2010. The green Solow model. *J. Econ. Growth* 15, 127–153.
- Brock, W.A., Xepapadeas, A., 2018. Modeling coupled climate, ecosystems, and economic systems. In: Dasgupta, P., Pattanayak, S.K., Smith, V.K. (Eds.), *Handbook of Environmental Economics*, vol. 4. Elsevier, Amsterdam, pp. 1–60.
- Dietz, S., Venmans, F., 2019. The endowment effect, discounting and the environment. *J. Environ. Econ. Manag.* 97, 67–91.
- Drupp, M.A., 2018. Limits to substitution between ecosystem services and manufactured goods and implications for social discounting. *Environ. Resour. Econ.* 69, 135–158.

- Drupp, M.A., Hänsel, M.C., 2018. Relative Prices and Climate Policy: How the Scarcity of Non-market Goods Drives Policy Evaluation. *Economics Working Paper* 2018-01. University of Kiel.
- Fisher, A., Krutilla, J., 1975. Resource conservation, environmental preservation, and the rate of discount. *Q. J. Econ.* 89, 358–370.
- Gollier, C., 2002. Discounting an uncertain future. *J. Public Econ.* 85, 149–166.
- Gollier, C., 2010. Ecological discounting. *J. Econ. Theory* 145, 812–829.
- Gollier, C., 2013. *Pricing the Planet's Future: the Economics of Discounting in an Uncertain World*. Princeton University Press, Princeton NJ.
- Gollier, C., Hammitt, J., 2014. The long-run discount rate controversy. *Annual Review of Resource Economics* 6, 273–295.
- Groom, B., Hepburn, C., 2017. Looking back at social discounting policy: the influence of papers, presentations, political preconditions, and personalities. *Rev. Environ. Econ. Policy* 11, 336–356.
- Guesnerie, R., 2004. Calcul économique et développement durable. *Rev. Econ.* 55, 163–382.
- Hoel, M., Sterner, T., 2007. Discounting and relative prices. *Clim. Change* 84, 265–280.
- Jones, C.I., 2016. The facts of economic growth. In: Taylor, J.B., Uhlig, H. (Eds.), *Handbook of Macroeconomics* 2A. Elsevier, Amsterdam, pp. 3–69.
- Koetse, M.J., Renes, G., Ruijs, A., de Zeeuw, A.J., 2018. Relative Price Increase for Nature and Ecosystem Services in Cost-Benefit Analysis. PBL Netherlands Environmental Assessment Agency, PBL publication number: 3214.
- Markandya, A., Pedrosa-Galinato, S., 2007. How substitutable is natural capital? *Environ. Resour. Econ.* 37, 297–312.
- Newell, R.G., Pizer, W.A., 2003. Discounting the distant future: how much do uncertain rates increase valuations? *J. Environ. Econ. Manag.* 46, 52–71.
- Nordhaus, W.D., 2008. *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven CT.
- Quaas, M.F., Bröcker, J., 2016. Substitutability and the Social Cost of Carbon in a Solvable Growth Model with Irreversible Climate Change. *Economics Working Paper* 2016-09. University of Kiel.
- Stern, N.H., 2006. *The Economics of Climate Change*. H.M. Treasury, London.
- Sterner, T., Persson, U.M., 2008. An even Sterner review: introducing relative prices into the discounting debate. *Rev. Environ. Econ. Policy* 2, 61–76.
- Traeger, C.P., 2011. Sustainability, limited substitutability, and non-constant social discount rates. *J. Environ. Econ. Manag.* 62, 215–228.
- Weikard, H.-P., Zhu, X., 2005. Discounting and environmental quality: when should dual rates be used? *Econ. Modell.* 22, 868–878.
- Weitzman, M.L., 1994. On the “environmental” discount rate. *J. Environ. Econ. Manag.* 26, 200–209.
- Weitzman, M.L., 2007. Subjective expectations and asset-return puzzles. *Am. Econ. Rev.* 97, 1102–1130.
- Weitzman, M.L., 2010. Risk-adjusted gamma discounting. *J. Environ. Econ. Manag.* 60, 1–13.
- Werf, E. van der, 2008. Production functions for climate policy modeling: an empirical analysis. *Energy Econ.* 30, 2964–2979.